5.0 MATHEMATICS (121)

This Mathematics report is based on an analysis of performance of candidates who sat the year 2009 KCSE Mathematics examinations. Candidates’ abilities were tested in two papers. **Paper 1 (121/1) and paper 2 (121/2).** The papers are equally weighted with each having two sections; Section 1 (50 marks) short answer questions of not more than four marks each and Section II (50 marks), a choice of eight questions of 10 marks each where candidates answer any five.

Paper 1 (121/1) tests mainly Forms 1 and 2 work while Paper 2 (121/2) tests mainly forms 3 and 4 work.

It is hoped that this report will be helpful to teachers in the teaching/learning process as well as in preparing candidates for future examinations.

5.1 CANDIDATES’ GENERAL PERFORMANCE

The table below shows the overall performance for both papers in the last four years.

**Table 10: Candidates’ Overall Performance in Mathematics for the last four years**

<table>
<thead>
<tr>
<th>Year</th>
<th>Paper</th>
<th>Candidature</th>
<th>Maximum Score</th>
<th>Mean Score</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>1</td>
<td>238684</td>
<td>100</td>
<td>22.71</td>
<td>20.09</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>100</td>
<td>15.36</td>
<td>15.97</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td></td>
<td>200</td>
<td><strong>38.08</strong></td>
<td><strong>35.00</strong></td>
</tr>
<tr>
<td>2007</td>
<td>1</td>
<td>273504</td>
<td>100</td>
<td>19.55</td>
<td>19.09</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>100</td>
<td>19.91</td>
<td>20.74</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td></td>
<td>200</td>
<td><strong>39.46</strong></td>
<td><strong>39.83</strong></td>
</tr>
<tr>
<td>2008</td>
<td>1</td>
<td>304908</td>
<td>100</td>
<td>22.76</td>
<td>22.76</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>100</td>
<td>19.82</td>
<td>19.56</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td></td>
<td>200</td>
<td><strong>42.59</strong></td>
<td><strong>41.53</strong></td>
</tr>
<tr>
<td>2009</td>
<td>1</td>
<td>335615</td>
<td>100</td>
<td>22.37</td>
<td>19.71</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>100</td>
<td>19.89</td>
<td>18.78</td>
</tr>
<tr>
<td></td>
<td>Overall</td>
<td></td>
<td>200</td>
<td><strong>42.26</strong></td>
<td><strong>37.65</strong></td>
</tr>
</tbody>
</table>

From the table the following observations can be made:

5.1.1 The overall performance in Mathematics has slightly declined from a mean of 42.59 in year 2008 to 42.26 in year 2009.

5.1.2 There is a slight improvement in the performance of Paper 2 (121/2) from a mean of 19.82 in year 2008 to a mean of 19.89 in year 2009. However, there is decline in Paper 1 (121/1) from a mean of 22.76 in year 2008 to a mean of 22.37 in year 2009.

5.1.3 There has been a significant increase in the candidature over the years.

5.2 INDIVIDUAL QUESTION ANALYSIS

Questions in which candidates’ performance was poor have been identified and are analysed in the following discussion.

5.2.1 Paper 1 (121/1)

**Question 3**

Given that the ratio \( x : y = 2 : 3 \), find the ratio \( (5x - 2y) : (x + y) \)

The question tested on candidates’ knowledge of ratios given one ratio then find another.

22
Weaknesses
Candidates did not seem to understand the meaning of ratios. They used \(x:y = 2:3\) to mean \(x=2\) and \(y=3\), which is a misconception.

Expected Response
\[
x : y = 2 : 3 \Rightarrow \frac{x}{y} = \frac{2k}{3k}
\]
(where \(k\) is a constant)
\[
x = 2k, \quad y = 3k
\]
The thus
\[
(5x - 2y) : (x + y)
\]
\[
= (5 \cdot 2k - 2 \cdot 3k) : (2k + 3k)
\]
\[
= (10 - 6)k : 5k
\]
\[
= 4 : 5
\]

Advice to Teachers
Teachers are expected to be thorough when teaching ratios and ratio proportions. Give more general examples to erase the misconceptions that if \(x:y = 2:3\) then \(x=2\) and \(y=3\).

Question 4
A bus travelling at an average speed of 63 km/h left a station at 8.15 a.m. A car later left the same station at 9.00 a.m. and caught up with the bus at 10.45 a.m. Find the average speed of the car.

This question tested on relative motion between two vehicles moving in the same direction.

Weaknesses
Candidates had difficulty interpreting the relative speed.

Expected Response
Distance covered by bus
\[
= 63 \times (10.45 - 8.15)
\]
\[
= 63 \times 2.5
\]
\[
= 157.5
\]

Speed of car
\[
= \frac{157.5}{1.75}
\]
\[
= 90 \text{ km/h}
\]

Advice to Teachers
Take more time teaching this topic and explore different scenarios e.g. relative motion of bodies moving in the same direction and relative motion of bodies moving in opposite directions.

Question 11
Line \(AB\) shown below is a side of a trapezium \(ABCD\) in which angle \(ABC = 105^\circ\), \(BC = 4 \text{ cm},\)
\(CD = 5 \text{ cm}\) and \(CD\) is parallel to \(AB\).

Using a ruler and a paid of compasses only:
(a) complete the trapezium
(b) locate point \(T\) on line \(AB\) such that angle \(ATD = 90^\circ\).
This question tested on basic construction of angle 105°, dropping a perpendicular from a point to a line and construction of parallel lines.

**Weaknesses**
Candidates used protractor and set square, contrary to instructions.

**Expected response**

![Diagram](image)

(a) Construction of 105°
Fixing point C and construction of line parallel to AB through C
Completion of trapezium ABCD

(b) Location of point T

**Advice to Teachers**
Advise students to do more practice on construction using a ruler and pair of compasses only.

**Question 16**
The following data was obtained for the masses of certain animals.

<table>
<thead>
<tr>
<th>Mass (x kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 ≤ x &lt; 5.5</td>
<td>16</td>
</tr>
<tr>
<td>5.5 ≤ x &lt; 7.5</td>
<td>20</td>
</tr>
<tr>
<td>7.5 ≤ x &lt; 13.5</td>
<td>18</td>
</tr>
<tr>
<td>13.5 ≤ x &lt; 15.5</td>
<td>14</td>
</tr>
</tbody>
</table>

Complete the histogram on the grid provided below.

![Histogram](image)

The question required construction of Histogram using frequency density.
Weaknesses
Most candidates seemed not to understand the concept of frequency density.

Expected Response

![Histogram](Image)

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 - 5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>5.5 - 7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>7.5 - 13.5</td>
<td>13.5</td>
</tr>
</tbody>
</table>

Advice to Teachers
Teach the topic on drawing of histograms when the classes are of unequal width i.e. by use of frequency density.

Question 19
A school planned to buy x calculators for a total of Ksh 16 200. The supplier agreed to offer a discount of Ksh 60 per calculator. The school was then able to get three extra calculators for the same amount of money.

(a) Write an expression in terms of x, for the:
   (i) original price of each calculator;
   (ii) price of each calculator after the discount.

(b) Form an equation in x and hence determine the number of calculators the school bought.

(c) Calculate the discount offered to the school as a percentage.

The question tested skill on formation of quadratic equations and solving them given a word problem.

Weaknesses
Most candidates interpreted the question wrongly and hence forming the equation was difficult.

Expected Response

(a) (i) Original Price = \( \frac{16200}{x} \)

   (ii) Price after discount = \( \frac{16200}{x + 3} \)

(b) (i) \[ \frac{16200}{x} - 60 = \frac{16200}{x + 3} \]

   \[ \Rightarrow \frac{16200 - 60x}{x} = \frac{16200}{x + 3} \]
\[
\Rightarrow (16200 - 60x)(x + 3) = 16200x
\]

\[
16200x + 16200 \times 3 - 60x^2 - 180x = 16200x
\]

\[
60x^2 + 180x - 48600 = 0
\]

\[
x^2 + 3x - 810 = 0
\]

\[
(x + 30)(x - 27) = 0
\]

\[x = -30 \text{ or } x = 27\]

No. of calculators bought = 30

(c) Initial cost of calculators
\[
\frac{16200}{27} = 600
\]

Discount offered as a percentage

\[
\frac{16200}{27} - \frac{16200}{30} \times 100 = 10\%\]

Advice to Teachers
Wide exposure of candidates to many problems of this nature will help the candidates to grasp the concept quite well.

Question 22
The diagram below shows the speed-time graph for a train travelling between two stations. The train starts from rest and accelerates uniformly for 150 seconds. It then travels at a constant speed for 300 seconds and finally decelerates uniformly for 200 seconds.

Given that the distance between the two stations is 104500m calculate:
(a) maximum speed, in km/h the train attained;
(b) acceleration;
(c) distance the train travelled during the last 100 seconds;
(d) time the train takes to travel the first half of the journey.

This question tested on linear motion. Candidates were required to calculate speed, acceleration, distance and time.

Weaknesses
Most candidates could not interpret the question correctly, thus found it difficult to answer well.
Expected Response

(a) \[ \frac{1}{2} \times 150h + \frac{1}{2} \times 200h + 300h + 10450 \]
\[ 475h = 10450 \]
\[ h = \frac{22 \times 60 \times 60}{1000} = 79.2 \text{ km/h} \]

(b) Acceleration
\[ \frac{22 \text{ m/s}}{150s} = \frac{11}{75} \text{ m/s}^2 \text{ or } 0.1467 \text{ m/s}^2 \]

(c) \[ \frac{1}{2} \times 100 \times 11 = 550 \]

(d) Time for half of journey
\[ \frac{1}{2} \times 22(150 + t + t) = \frac{1}{2} \times 10450 \]
\[ t = 162.5 \]
Total time \[ = 150 + 162.5 = 312.5 \]

Advice to Teachers
Analysis of graphs in linear motion is quite important for deeper understanding of the concept.

5.2.2 PAPER 2 (121/2)

Question 2
Find a quadratic equation whose roots are \( 1.5 + \sqrt{2} \) and \( 1.5 - \sqrt{2} \), expressing it is the form \( ax^2 + bx + c = 0 \), where a, b and c are integers.

This question tested on formation of quadratic equations given roots. The roots were in the form of surds.

Weaknesses
Types of roots seem not familiar to the candidates.
Expected Response
\[ (x - 1.5 - \sqrt{2})(x - 1.5 + \sqrt{2}) = 0 \]
\[ x^2 - 1.5x + x\sqrt{2} - 1.5x + 2.25 - 1.5\sqrt{2} - x\sqrt{2} + 1.5\sqrt{2} - 2 = 0 \]
\[ 4x^2 - 12x + 1 = 0 \]

Advice to Teachers
Emphasis on teaching of surds and use them in different situations not only in their simplification.

Question 4
In the figure below, O is the centre of the circle and radius ON is perpendicular to the line TS at N.

Using a ruler and a pair of compasses only, construct a triangle ABC to inscribe the circle, given that angle ABC = 60°, BC = 12 cm and points B and C are on the line TS.
This question tested on construction of a triangle to inscribe the circle.

Weaknesses
Candidates failed to get the properties of the angles in an inscribed circle. Thus were unable to construct the triangle.

Expected Responses

Advice to Teachers
Teach the properties of angles, in an inscribed circle.

Question 7
In a certain commercial bank, customers may withdraw cash through one of the two tellers at the counter. On average, one teller takes 3 minutes while the other teller takes 5 minutes to serve a customer. If the two tellers start to serve the customers at the same time, find the shortest time it takes to serve 200 customers.

This question tested on application of LCM to real life situations.

Weaknesses
Candidates failed to understand that it was LCM being tested.

Expected Response
The LCM of 3 and 5 = 15
In 15 minutes, 8 customers will be served
\[ \therefore \text{ total time} = \frac{200}{8} \times 15 \]
\[ = 375 \text{ min.} \]
Advice to Teachers
When teaching LCM and GCD relate them to application in real life situations.

Question 13
Point P(40°S, 45°E) and point Q(40°S and 60°W) are on the surface of the Earth. Calculate the shortest distance along a circle of latitude between the two points.

This question tested on distance along the Earth’s surface. Candidates were expected to calculate the shortest distance along a circle of latitude between two points.

Weaknesses
Most candidates did not know the concept. Hence could not answer correctly.

Expected Response
Longitude difference \(45^0 + 60^0 = 105^0\)
Distance in km
\[
\frac{105}{360} \times 2 \times 3.142 \times 6370 \cos 40^0
\]
8943.7 km

Advice to Teachers
Teach the topic thoroughly.

Question 16
A particle moves in a straight line with a velocity \(\dot{v}\) m s\(^{-1}\). Its velocity after \(t\) seconds is given by
\[\dot{v} = 3t^2 - 6t - 9.\]
The figure below is a sketch of the velocity-time graph of the particle.

\[
\begin{align*}
\text{Calculate the distance the particle moves between } t = 1 \text{ and } t = 4. \\
\text{This question tested on determination of distance using integration.}
\end{align*}
\]

Weaknesses
Most candidates did not separate the regions below and region above the horizontal axis.

Expected Response
\[
\begin{align*}
\int \left[3t^2 - 6t - 9\right] dt &= t^3 - 3t^2 - 9t + c \\
\left[t^3 - 3t^2 - 9t\right]_1^4 &= \left[\frac{1}{3} - 3(3) - 9(3)\right] - \left[\frac{1}{3} - 3(1) - 9(1)\right] \\
&= -16 \\
\left[t^3 - 3t^2 - 9t\right]_{-2}^1 &= \left[\frac{1}{3} - 3(4) - 9(4)\right] - \left[\frac{1}{3} - 3(3) - 9(3)\right] \\
&= 7
\end{align*}
\]
Distance travelled \(\approx 16 + 7 = 23\) m
Question 19
The table below shows the number of goals scored in handball matches during a tournament.

<table>
<thead>
<tr>
<th>Number of goals</th>
<th>0-9</th>
<th>10-19</th>
<th>20-29</th>
<th>30-39</th>
<th>40-49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td>2</td>
<td>14</td>
<td>24</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Draw a cumulative frequency curve on the grid provided.

(b) Using the curve drawn in (a) above determine:
(i) the median
(ii) the number of matches in which goals scored were not more than 37.
(iii) the inter-quartile range.

The question tested on drawing an ogive. Candidates were expected to determine the class boundaries then plot the ogive on the grid and use it to answer questions that followed.

Weaknesses
Candidates did not determine the lower limit of the first class. Candidates had difficulty in inter quartile range.

Expected Response

(b)(i) Median goals: $25.5 \pm 0.5$

(ii) number of matches in which scores were between $0 \& 37 \pm 49$

(iii) $Q_1 = 19 \pm 0.5$

$Q_3 = 33 \pm 0.5$

Inter quartile range 33-19-14
Advice to Teachers
Explain the starting point when the first class starts with zero. Teach thoroughly on ogive and interpretation of interquartile range using the ogive.

Question 22
The figure below shows a right pyramid mounted onto a cuboid. AB = BC = 15\sqrt{2} \text{ cm}, CG = 8 \text{ cm}, and VG = 17\sqrt{2} \text{ cm}.

![Diagram of a right pyramid mounted onto a cuboid]

Calculate:
(a) the length of AC,
(b) the angle between the line AG and the plane ABCD;
(c) the vertical height of point V from the plane ABCD;
(d) the angle between the planes EFV and ABCD.

This was a question in 3-Dimension Geometry. It required candidates to use knowledge of Pythagoras theorem and trigonometry to solve the problem.

Weaknesses
Identifying the required angle was a problem.

Expected Responses
(a) \( AC = \sqrt{(15\sqrt{2})^2 + (15\sqrt{2})^2} = 30\text{ cm} \)

(b) Identification of \( \theta \)
\[ \tan \theta = \frac{8}{30} \text{ or equivalent} \]
\[ \theta = 14.93^\circ \]

(c) Pyramid height \( \sqrt{(17\sqrt{2})^2 - 15^2} \)
\[ = 18.79 \text{ cm} \]
\[ VO = 18.79 + 8 \]
\[ = 26.79 \text{ cm} \]

(d) Identification of \( \alpha \)
\[ \tan \alpha = \frac{18.79}{7.5\sqrt{2}} \]
\[ \alpha = 60.55^\circ \]
Advice to Teachers
Use models to demonstrate angles between planes.

Question 24
Amina carried out an experiment to determine the average volume of a ball bearing. She started by submerging three ball bearings in water contained in a measuring cylinder. She then added one ball at a time into the cylinder until the balls were nine.

The corresponding readings were recorded as shown in the table below:

<table>
<thead>
<tr>
<th>Number of ball bearings</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring cylinder reading (F)</td>
<td>98.0</td>
<td>105.0</td>
<td>123.0</td>
<td>130.5</td>
<td>145.6</td>
<td>156.9</td>
<td>170.0</td>
</tr>
</tbody>
</table>

(a)  
(i) On the grid provided, plot (x, y) where x is the number of ball bearings and y is the corresponding measuring cylinder reading.
(ii) Use the plotted points to draw the line of best fit.

(b)  
Use the line of best fit to determine:
(i) the average volume of a ball bearing;
(ii) the equation of the line.

(c) Using the equation of the line in b(ii) above, determine the volume of the water in the cylinder.

The question tested on drawing line of best fit.

Weaknesses
Poor choice of scale, candidates used points from table to locate line.

Expected Response

(a)  
(i) Scale
   Plotting

(ii) Line of best fit

(b)  
(i) Average volume of ball bearing
\[
\frac{133 - 108}{6 - 4} = 12.5
\]

(ii) \[\begin{align*}
\frac{y - 133}{x - 6} &= 12.5 \\
y &= 12.5x + 58
\end{align*}\]

(c) Volume of water in cylinder is the volume of y when \(x = 0\)
\[y = 12.5 \times 0 + 58\]
\[= 58\]

**Advice to Teachers**
Emphasize on qualities of the line of best fit.

### 5.3 General Comments

5.3.1 Teachers are advised to cover the syllabus early enough and have time for comprehensive revision with the students.

5.3.2 Candidates should read and adhere to the instructions given in the examination e.g. if told to use ruler and pair of compasses only do not use a protractor or set square.

5.3.3 Emphasis on application of the concept learnt in class to real life situations should be done.

5.3.4 Teachers should encourage candidates to attempt only the required questions, they should avoid leaving blanks especially in section I.
29.3 MATHEMATICS (121)

29.3.1 Mathematics Paper 1 (121/1)

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. Without using mathematical tables or calculators, evaluate \( \frac{\sqrt{5184}}{6 \times 18 + 9 + (5 - 3)} \).

2. Without using a calculator, evaluate, \( \frac{2 \frac{1}{4} + \frac{1}{2} + \frac{5}{6} \text{ of } 2 \frac{2}{3}}{1 \frac{3}{10}} \), leaving the answer as a fraction in its simplest form.

3. Given that the ratio \( x : y = 2 : 3 \), find the ratio \( (5x - 2y) : (x + y) \).

4. A bus travelling at an average speed of 63 km/h left a station at 8.15 a.m. A car later left the same station at 9.00 a.m. and caught up with the bus at 10.45 a.m. Find the average speed of the car.

5. Without using logarithm tables or calculators, evaluate, \( \frac{64^{-\frac{1}{3}} \times 27000^3}{2^{-4} \times 3^6 \times 5^2} \).

6. The figure below represents a plot of land ABCD such that \( AB = 85 \text{ m}, BC = 75 \text{ m}, CD = 60 \text{ m}, DA = 50 \text{ m} \) and angle ACB = 90°.

![Diagram of plot ABCD](image)

Determine the area of the plot in hectares correct to two decimal places.

7. A watch which loses a half-minute every hour was set to read the correct time at 05 45 h on Monday. Determine the time, in the 12-hour system, the watch will show on the following Friday at 19 45 h.

8. Simplify the expression \( \frac{12x^2 + ax - 6a^2}{9x^2 - 4a^2} \).
9. A line which joins the points A (3, k) and B (2, 5) is parallel to another line whose equation is $5y + 2x = 10$.
Find the value of k. (3 marks)

10. The size of an interior angle of a regular polygon is $6\frac{1}{2}$ times that of its exterior angle.
Determine the number of sides of the polygon. (3 marks)

11. Line AB shown below is a side of a trapezium ABCD in which angle ABC = 105°, BC = 4 cm, CD = 5 cm and CD is parallel to AB.

Using a ruler and a pair of compasses only:
(a) complete the trapezium; (3 marks)
(b) locate point T on line AB such that angle ATD = 90°. (1 mark)

12. An electric pole is supported to stand vertically on a level ground by a tight wire. The wire is pegged at a distance of 6 metres from the foot of the pole as shown.

The angle which the wire makes with the ground is three times the angle it makes with the pole.
Calculate the length of the wire to the nearest centimetre. (3 marks)

13. Solve the equation: $\sin(3x + 30°) = \frac{\sqrt{3}}{2}$, for $0° \leq x \leq 90°$. (4 marks)
14. The diagonals of a rhombus PQRS intersect at T. Given that P(2, 2), Q(3, 6) and R(−1, 5):
(a) draw the rhombus PQRS on the grid provided; (1 mark)

(b) state the coordinates of T. (1 mark)

15. Abdi sold a radio costing Ksh 3 800 at a profit of 20%. He earned a commission of 22½% on the profit. Find the amount he earned. (2 marks)

16. The following data was obtained for the masses of certain animals.

<table>
<thead>
<tr>
<th>Mass (x kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 ≤ x &lt; 5.5</td>
<td>16</td>
</tr>
<tr>
<td>5.5 ≤ x &lt; 7.5</td>
<td>20</td>
</tr>
<tr>
<td>7.5 ≤ x &lt; 13.5</td>
<td>18</td>
</tr>
<tr>
<td>13.5 ≤ x &lt; 15.5</td>
<td>14</td>
</tr>
</tbody>
</table>

Complete the histogram on the grid provided below. (3 marks)
17 In the figure below (not drawn to scale). \( AB = 8 \text{ cm}, AC = 6 \text{ cm}, AD = 7 \text{ cm}, CD = 2.82 \text{ cm} \) and angle \( CAB = 50^\circ \).

![Diagram of triangle ABC with lengths and angles labeled]

Calculate, to 2 decimal places:

(a) the length BC; 

(b) the size of angle ABC; 

(c) the size of angle CAD; 

(d) the area of triangle ACD.

18 The marks scored by a group of pupils in a mathematics test were as recorded in the table below.

<table>
<thead>
<tr>
<th>Marks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 9</td>
<td>1</td>
</tr>
<tr>
<td>10 - 19</td>
<td>2</td>
</tr>
<tr>
<td>20 - 29</td>
<td>4</td>
</tr>
<tr>
<td>30 - 39</td>
<td>7</td>
</tr>
<tr>
<td>40 - 49</td>
<td>10</td>
</tr>
<tr>
<td>50 - 59</td>
<td>16</td>
</tr>
<tr>
<td>60 - 69</td>
<td>20</td>
</tr>
<tr>
<td>70 - 79</td>
<td>6</td>
</tr>
<tr>
<td>80 - 89</td>
<td>3</td>
</tr>
<tr>
<td>90 - 99</td>
<td>1</td>
</tr>
</tbody>
</table>

(a)  
(i) State the modal class. 

(ii) Determine the class in which the median mark lies.

(b) Using an assumed mean of 54.4, calculate the mean mark.
A school planned to buy $x$ calculators for a total cost of Ksh 16 200. The supplier agreed to offer a discount of Ksh 60 per calculator. The school was then able to get three extra calculators for the same amount of money.

(a) Write an expression in terms of $x$, for the:
   
   (i) original price of each calculator;  
   (ii) price of each calculator after the discount.

(b) Form an equation in $x$ and hence determine the number of calculators the school bought.

(c) Calculate the discount offered to the school as a percentage.

---

The position vectors of points $A$ and $B$ with respect to the origin $O$, are $(-8, 5)$ and $(12, -5)$ respectively. Point $M$ is the midpoint of $AB$ and $N$ is the midpoint of $OA$.

(a) Find:
   
   (i) the coordinates of $N$ and $M$;  
   (ii) the magnitude of $NM$.

(b) Express vector $NM$ in terms of $OB$.

(c) Point $P$ maps onto $P'$ by a translation $(-5, 8)$. Given that $OP - OM = 2MN$, find the coordinates of $P'$.

---

A glass, in the form of a frustum of a cone, is represented by the diagram below. The glass contains water to a height of 9 cm. The bottom of the glass is a circle of radius 2 cm while the surface of the water is a circle of radius 6 cm.

(a) Calculate the volume of the water in the glass.

(b) When a spherical marble is submerged into the water in the glass, the water level rises by 1 cm. Calculate:
   
   (i) the volume of the marble;  
   (ii) the radius of the marble.

---

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The diagram below shows the speed-time graph for a train travelling between two stations. The train starts from rest and accelerates uniformly for 150 seconds. It then travels at a constant speed for 300 seconds and finally decelerates uniformly for 200 seconds.

Given that the distance between the two stations is 10,450 m, calculate the:

(a) maximum speed, in km/h, the train attained; 

(b) acceleration; 

(c) distance the train travelled during the last 100 seconds; 

(d) time the train takes to travel the first half of the journey.

(3 marks)

(2 marks)

(2 marks)

(3 marks)

23 Three points P, Q and R are on a level ground. Q is 240 m from P on a bearing of 230°. R is 120 m to the east of P.

(a) Using a scale of 1 cm to represent 40 m, draw a diagram to show the positions of P, Q and R in the space provided below.

(b) Determine:

(i) the distance of R from Q; 

(ii) the bearing of R from Q.

(2 marks)

(2 marks)

(2 marks)

(c) A vertical post stands at P and another one at Q. A bird takes 18 seconds to fly directly from the top of the post at Q to the top of the post at P.

Given that the angle of depression of the top of the post at P from the top of the post at Q is 9°, calculate:

(i) the distance, to the nearest metre, the bird covers; 

(ii) the speed of the bird in km/h.

(2 marks)

(2 marks)
24. (a) On the grid provided, draw a graph of the function

\[ y = \frac{1}{2} x^2 - x + 3 \] for \( 0 \leq x \leq 6. \] (3 marks)

(b) Calculate the mid-ordinates for 5 strips between \( x = 1 \) and \( x = 6 \), and hence use the mid-ordinate rule to approximate the area under the curve between \( x = 1 \), \( x = 6 \) and the \( x \)-axis. (3 marks)

(c) Assuming that the area determined by integration to be the actual area, calculate the percentage error in using the mid-ordinate rule. (4 marks)
29.3.2 Mathematics Paper 1 (121/2)  

SECTION 1 (50 marks)

Answer all the questions in this section in the spaces provided.

1. A farmer feeds every two cows on 480 Kg of hay for four days. The farmer has 20160 Kg of hay which is just enough to feed his cows for 6 weeks. Find the number of cows in the farm. (3 marks)

2. Find a quadratic equation whose roots are $1.5 + \sqrt{2}$ and $1.5 - \sqrt{2}$, expressing it in the form $ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are integers. (3 marks)

3. The mass of a wire in grams (g) is partly a constant and partly varies as the square of its thickness $t$ mm. When $t = 2$ mm, $m = 40$ g and when $t = 3$ mm, $m = 65$ g.
Determine the value of $m$ when $t = 4$ mm. (4 marks)

4. In the figure below, O is the centre of the circle and radius ON is perpendicular to the line TS at N.

![Diagram of a circle with O as the center, N on the circumference, and ON perpendicular to TS at N.]

Using a ruler and a pair of compasses only, construct a triangle ABC to inscribe the circle, given that angle ABC = 60°, BC = 12 cm and points B and C are on the line TS. (4 marks)

5. A solution was gently heated, its temperature readings taken at intervals of 1 minute and recorded as shown in the table below.

<table>
<thead>
<tr>
<th>Time (Min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>4</td>
<td>5.2</td>
<td>8.4</td>
<td>14.3</td>
<td>16.8</td>
<td>17.5</td>
</tr>
</tbody>
</table>

(a) Draw the time-temperature graph on the grid provided. (2 marks)

(b) Use the graph to find the average rate of change in temperature between $t = 1.8$ and $t = 3.4$. (2 marks)
Vector $\mathbf{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{OB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$. Point $C$ is on $\mathbf{OB}$ such that $\mathbf{CB} = 2\mathbf{OC}$ and point $D$ is on $\mathbf{AB}$ such that $\mathbf{AD} = 3\mathbf{DB}$.

Express $\mathbf{CD}$ as a column vector. (3 marks)

7. In a certain commercial bank, customers may withdraw cash through one of the two tellers at the counter. On average, one teller takes 3 minutes while the other teller takes 5 minutes to serve a customer. If the two tellers start to serve the customers at the same time, find the shortest time it takes to serve 200 customers. (3 marks)

8. (a) Expand and simplify the binomial expression $(2 - x)^2$ in ascending powers of $x$: (2 marks)

(b) Use the expansion up to the fourth term to evaluate $(1.97)^3$ correct to 4 decimal places. (2 marks)

9. The area of triangle $\text{FGH}$ is 21 cm$^2$. The triangle $\text{FGH}$ is transformed using the matrix $\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$. Calculate the area of the image of triangle $\text{FGH}$. (2 marks)

10. Simplify $\frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}}$. (2 marks)

11. A circle whose equation is $(x - 1)^2 + (y - k)^2 = 10$ passes through the point $(2, 5)$. Find the coordinates of the two possible centres of the circle. (3 marks)

12. On a certain day, the probability that it rains is $\frac{1}{4}$. When it rains the probability that Omondi carries an umbrella is $\frac{2}{3}$. When it does not rain the probability that Omondi carries an umbrella is $\frac{1}{6}$. Find the probability that Omondi carried an umbrella that day. (2 marks)

13. Point $P(40^\circ S, 45^\circ E)$ and point $Q(40^\circ S, 60^\circ W)$ are on the surface of the Earth. Calculate the shortest distance along a circle of latitude between the two points. (3 marks)

14. Solve $4 - 4 \cos^2 \alpha = 4 \sin \alpha - 1$ for $0^\circ \leq \alpha \leq 360^\circ$. (4 marks)
15 In the figure below, AT is a tangent to the circle at A. Angle $\angle ATB = 48^\circ$, $BC = 5$ cm and $CT = 4$ cm.

Calculate the length AT. \hspace{1cm} (2 marks)

16 A particle moves in a straight line with a velocity $V$ m/s$^{-1}$. Its velocity after $t$ seconds is given by $V = 3t^2 - 6t - 9$.

The figure below is a sketch of the velocity-time graph of the particle.

Calculate the distance the particle moves between $t = 1$ and $t = 4$. \hspace{1cm} (4 marks)

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17 A water vendor has a tank of capacity 18 900 litres. The tank is being filled with water from two pipes A and B which are closed immediately when the tank is full. Water flows at the rate of 150 000 cm$^3$/minute through pipe A and 120 000 cm$^3$/minute through pipe B.

(a) If the tank is empty and the two pipes are opened at the same time, calculate the time it takes to fill the tank. \hspace{1cm} (3 marks)

(b) On a certain day the vendor opened the two pipes A and B to fill the empty tank. After 25 minutes he opened the outlet tap to supply water to his customers at an average rate of 20 litres per minute.

(i) Calculate the time it took to fill the tank on that day. \hspace{1cm} (4 marks)
(ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6300 litres, calculate, in litres, the amount of water that was wasted.

(3 marks)

(ii) The vendor supplied a total of 542 jerricans, each containing 25 litres of water, on that day. If the water that remained in the tank was 6300 litres, calculate, in litres, the amount of water that was wasted.

(3 marks)

At the beginning of the year 1998, Kanyingi bought two houses, one in Thika and the other one in Nairobi, each at Ksh 1 240 000. The value of the house in Thika appreciated at the rate of 12% p.a.

(a) Calculate the value of the house in Thika after 9 years, to the nearest shilling.

(2 marks)

(b) After n years, the value of the house in Thika was Ksh 2 741 245 while the value of the house in Nairobi was Ksh 2 917 231.

(i) Find n.

(4 marks)

(ii) Find the annual rate of appreciation of the house in Nairobi.

(4 marks)
The table below shows the number of goals scored in handball matches during a tournament.

<table>
<thead>
<tr>
<th>Number of goals</th>
<th>0 - 9</th>
<th>10 - 19</th>
<th>20 - 29</th>
<th>30 - 39</th>
<th>40 - 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td>2</td>
<td>14</td>
<td>24</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Draw a cumulative frequency curve on the grid provided. (5 marks)

(b) Using the curve drawn in (a) above determine:

(i) the median, (1 mark)

(ii) the number of matches in which goals scored were not more than 37; (1 mark)

(iii) the inter-quartile range. (3 marks)
Triangle PQR shown on the grid has vertices P(5, 5), Q(10, 10) and R(10, 15).

(a) Find the coordinates of the points P', Q' and R', the images of P, Q and R respectively under transformation M whose matrix is

\[
\begin{pmatrix}
-0.6 & 0.8 \\
0.8 & 0.6
\end{pmatrix}
\]

(2 marks)
(b) Given that M is a reflection:
   (i) draw triangle \( P'Q'R' \) and the mirror line of the reflection; 
   (ii) determine the equation of the mirror line of the reflection. 
   \( (2 \text{ marks}) \) \( (1 \text{ mark}) \)

(c) Triangle \( P''Q''R'' \) is the image of triangle \( P'Q'R' \) under reflection \( N \) where \( N \) is a reflection in the 
   \( y \)-axis.
   (i) Draw triangle \( P''Q''R'' \). 
   (ii) Determine a \( 2 \times 2 \) matrix equivalent to the transformation \( NM \). 
   \( (1 \text{ mark}) \) \( (2 \text{ marks}) \)
   (iii) Describe fully a single transformation that maps triangle \( PQR \) onto triangle \( P''Q''R'' \). 
   \( (2 \text{ marks}) \)

21 The table below shows income tax rates.

<table>
<thead>
<tr>
<th>Monthly income in Kenya shillings (Ksh)</th>
<th>Tax rate percentage (%) in each shilling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 9 680</td>
<td>10</td>
</tr>
<tr>
<td>From 9 681 to 18 800</td>
<td>15</td>
</tr>
<tr>
<td>From 18 801 to 27 920</td>
<td>20</td>
</tr>
<tr>
<td>From 27 921 to 37 040</td>
<td>25</td>
</tr>
<tr>
<td>From 37 041 and above</td>
<td>30</td>
</tr>
</tbody>
</table>

In a certain year, Robi's monthly taxable earnings amounted to Ksh 24 200.

(a) Calculate the tax charged on Robi’s monthly earnings. 
   \( (4 \text{ marks}) \)

(b) Robi was entitled to the following tax reliefs:
   I: monthly personal relief of Ksh 1 056;
   II: monthly insurance relief at the rate of 15% of the premium paid.
   Calculate the tax paid by Robi each month, if she paid a monthly premium of Ksh 2 400 towards 
   her life insurance policy. 
   \( (2 \text{ marks}) \)

(c) During a certain month, Robi received additional earnings which were taxed at 20% in each 
   shilling. Given that she paid 36.3% more tax that month, calculate the percentage increase in her 
   earnings. 
   \( (4 \text{ marks}) \)
22 The figure below shows a right pyramid mounted onto a cuboid. \( AB = BC = 15\sqrt{2} \, \text{cm}, \, \text{CG} = 8 \, \text{cm} \) and \( VG = 17\sqrt{2} \, \text{cm}\).

![Diagram of a right pyramid mounted onto a cuboid]

Calculate:

(a) the length of AC; (1 mark)

(b) the angle between the line AG and the plane ABCD; (3 marks)

(c) the vertical height of point \( V \) from the plane ABCD; (3 marks)

(d) the angle between the planes EFV and ABCD. (3 marks)

23 (a) The first term of an Arithmetic Progression (AP) is 2. The sum of the first 8 terms of the AP is 156.

(i) Find the common difference of the AP. (2 marks)

(ii) Given that the sum of the first \( n \) terms of the AP is 416, find \( n \). (2 marks)

(b) The 3\(^{rd}\), 5\(^{th}\) and 8\(^{th}\) terms of another AP form the first three terms of a Geometric Progression (GP).

If the common difference of the AP is 3, find:

(i) the first term of the GP; (4 marks)

(ii) the sum of the first 9 terms of the GP, to 4 significant figures. (2 marks)
Amina carried out an experiment to determine the average volume of a ball bearing. She started by submerging three ball bearings in water contained in a measuring cylinder. She then added one ball at a time into the cylinder until the balls were nine.

The corresponding readings were recorded as shown in the table below.

<table>
<thead>
<tr>
<th>Number of ball bearings (x)</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring cylinder reading (y)</td>
<td>98.0</td>
<td>105.0</td>
<td>123.0</td>
<td>130.5</td>
<td>145.6</td>
<td>156.9</td>
<td>170.0</td>
</tr>
</tbody>
</table>

(a) (i) On the grid provided, plot \((x, y)\) where \(x\) is the number of ball bearings and \(y\) is the corresponding measuring cylinder reading.  
(3 marks)

(ii) Use the plotted points to draw the line of best fit.  
(1 mark)

(b) Use the line of best fit to determine:
   (i) the average volume of a ball bearing;  
   (ii) the equation of the line.  
   (2 marks)

(c) Using the equation of the line in b(ii) above, determine the volume of the water in the cylinder.  
(2 marks)
30.3  MATHEMATICS (121)

30.3.1  Mathematics Paper 1 (121/1)

Q1  \[
\frac{\sqrt{5184}}{6 \times 18 + 9 + (5 - 3)}
\]
\[
= \frac{\sqrt{2^6 \times 3^4}}{6 \times 18 + 9 + 8}
\]
\[
= \frac{2^3 \times 3^2}{6 \times 2 + 8}
\]
\[
= \frac{72}{4}
\]
\[
= 18
\]

(3 marks)

Q2  \[
\frac{2 \frac{1}{4} + 3 \frac{3}{5} \div 5 \frac{2}{5}}{1 \frac{7}{10}}
\]
\[
= \frac{2 \frac{1}{4} + 3 \times 6 \div 2}{4 \frac{1}{5} \times 5 \div 2}
\]
\[
= \frac{2 \frac{1}{4} + 3 \times 1}{4 \frac{1}{5} \times 2}
\]
\[
= \frac{2 \frac{1}{4} + \frac{1}{2}}{4 \frac{7}{10}}
\]
\[
= \frac{2 \frac{1}{4}}{4 \frac{7}{10}}
\]
\[
= \frac{\frac{9}{4} \times 10}{\frac{47}{10}}
\]
\[
= \frac{90}{47}
\]
\[
= \frac{3}{2} \text{ or } 1 \frac{1}{2} \text{ or } 1.5
\]

(3 marks)

Q3  \[
x : y = 2 : 3 \Rightarrow \frac{x}{y} = \frac{2k}{3k}
\]

(\text{where } k \text{ is a constant})

\[
x = 2k, \quad y = 3k
\]
Thus \((5x - 2y) : (x + y)\)

\[
= \frac{5 \times 2k - 2 \times 3k}{2k + 3k} : (2k + 3k)
\]

\[
= \frac{10k - 6k}{5k}
\]

\[
= 4 : 5
\]

Q4 Distance covered by bus

\[
= 63 \times (10.45 - 8.15)
\]

\[
= 63 \times 2.5
\]

\[
= 157.5
\]

Speed of car

\[
= \frac{157.5}{1.75}
\]

\[
= 90 \text{ km/h}
\]

Q5

\[
\frac{64^{\frac{1}{2}} \times 27000^{\frac{2}{3}}}{2^{-4} \times 3^0 \times 5^2}
\]

\[
= \frac{1}{64^{\frac{1}{2}}} \times \frac{1}{2^{-4}} \times \frac{1}{3^0} \times \frac{1}{5^2}
\]

\[
= \frac{1}{8} \times \left(\frac{\sqrt{27000}}{25}\right)^2
\]

\[
= \frac{1}{8} \times \frac{900 \times 16}{25}
\]

\[
= 72
\]

(4 marks)
Q6 \[ AC = \sqrt{85^2 - 75^2} = \sqrt{1600} = 40 \]
Area of quad ABCD
\[
= \frac{1}{2} \times 40 \times 75 + \sqrt{75(75 - 60)(75 - 50)(75 - 40)}
\]
\[
= 1500 + \sqrt{984375}
\]
\[
= 1500 + 992
\]
\[
= 2492 \text{ m}^2
\]
\[
= 0.25 \text{ ha}
\]

(4 marks)

Q7 Time between Monday 0545 h and Friday 1945 h
\[
= 4 \times 24 + 14
\]
\[
= 110 \text{ h}
\]
Time lost \[ = 0.5 \times 110 \]
\[
= 55 \text{ min}
\]
\[
\therefore \text{ Time shown in 12-hour system} = 1945-55 = 1850 \text{ h} = 6.50 \text{ pm}
\]

(3 marks)

Q8 \[
\frac{12x^2 + ax - 6a^2}{9x^2 - 4a^2}
\]
\[
= \frac{(4x + 3a)(3x - 2a)}{(3x + 2a)(3x - 2a)}
\]
\[
= \frac{4x + 3a}{3x + 2a}
\]

(3 marks)

Q9 \[ y = \frac{-2}{5} x + 2 \]
\[
\therefore \text{ gradient} = \frac{-2}{5}
\]
\[
\frac{k - 5}{3 - 2} = \frac{2}{5}
\]
\[
k - 5 = -2 \]
\[
\Rightarrow k = 3
\]

(3 marks)

Q10 let exterior \( \angle \) (\( \angle \) at centre) be \( x^\circ \)
\[
\therefore 6.5x + x = 180
\]
\[
7.5x = 180
\]
\[
x = 24^\circ
\]
\[
\text{No. of sides} = \frac{360}{24} \\
= 15 \text{ sides}
\]

Q11

(a) - Construction of 105°
- Fixing point C and construction of parallel line AB through C
- Completion of trapezium ABCD

(b) Location of point T

Q12 Let angle between ground and wire be \(\theta^0\)

\[
\therefore \frac{1}{3} \theta = 90^0 \\
\Rightarrow \theta = 90 \times \frac{3}{4} = 67.5^0
\]

Let length of wire be \(x\) cm

\[
\therefore \cos 67.5 = \frac{6}{x}
\]

\[
x = \frac{6}{\cos 67.5} \Rightarrow 0.382683432
\]

\[
x = 15.68 \text{ m or } 1568 \text{ cm or } 15 \text{ m 68 cm}
\]

Q13 \[
\sin (3x + 30)^0 = \sin 60^0 \\
\sin (3x + 30)^0 = \sin 120^0 \\
3x + 30 = 60^0 \\
3x + 30 = 120^0 \\
\therefore x = 10^0; \ x = 30^0
\]

Q14

392
Rhombus PQRS

(b) Coordinates of T (0.5, 3.5)  

Q15 Commission earned
0.225 x 0.2 x 3800
= 171

Q16

Q17 (a) \( BC^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 50^\circ \)
\[
\sqrt{38.2912} = 6.19
\]

(b) Let \( \angle ABC = \beta^0 \)
\[
\frac{\sin \beta}{6} = \frac{\sin 50^0}{6.19}
\]
\[
\sin \beta = \frac{6 \sin 50^0}{6.19}
\]
\[
\therefore \beta = 47.95^0
\]

(c) Let \( \angle CAD = \alpha^0 \)
\[
2.82^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \cos \alpha
\]
\[
\cos \alpha = \frac{49 + 36 - 7.9524}{84}
\]
\[
\therefore \alpha = 23.48^0
\]

(d) Area \( \Delta ACD \)
\[
\frac{1}{2} \times 7 \times 6 \sin 23.48^0
\]
\[
\approx 8.37 \text{ cm}^2
\]

Q18 (a) (i) Model class = 60 – 69

(ii) Class where median mark lies

c.f 1
3
7
14
24
40
60
66
69
70

Class 50 – 59

<table>
<thead>
<tr>
<th>Class centres (x)</th>
<th>Fd</th>
<th>(d = x - A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>-49.9</td>
<td>-49.9</td>
</tr>
<tr>
<td>14.5</td>
<td>-79.8</td>
<td>-39.9</td>
</tr>
<tr>
<td>24.5</td>
<td>-119.6</td>
<td>-29.9</td>
</tr>
<tr>
<td>34.5</td>
<td>-139.3</td>
<td>-19.9</td>
</tr>
<tr>
<td>44.5</td>
<td>-99.0</td>
<td>-9.9</td>
</tr>
<tr>
<td>54.5</td>
<td>1.6</td>
<td>0.1</td>
</tr>
<tr>
<td>64.5</td>
<td>20.2</td>
<td>10.1</td>
</tr>
<tr>
<td>74.5</td>
<td>120.6</td>
<td>20.1</td>
</tr>
<tr>
<td>84.5</td>
<td>90.3</td>
<td>30.1</td>
</tr>
<tr>
<td>94.5</td>
<td>40.1</td>
<td>40.1</td>
</tr>
</tbody>
</table>

\[
\Sigma f = 70
\]
\[ \Sigma fd = -33 \]
\[ \therefore \text{Mean} = 54.4 + \frac{-33}{70} = 53.93 \]

(10 marks)

Q19  
(a)  
(i) Original Price \(\frac{16200}{x}\)  
(ii) Price after discount \(\frac{16200}{x+3}\)

(b)  
(i) \(\frac{16200}{x} - 60 = \frac{16200}{x+3}\)  
\[ \Rightarrow \frac{16200 - 60x}{x} = \frac{16200}{x+3} \]  
\[ \Rightarrow (16200 - 60x)(x+3) = 16200x \]

\[ 16200x + 16200 \times 3 - 60x^2 - 180x = 16200x \]

\[ 60x^2 + 180x - 48600 = 0 \]

\[ x^2 + 3x - 810 = 0 \]

\[ (x + 30)(x - 27) = 0 \]
\[ x = -30 \text{ or } x = 27 \]
No. of calculators bought = 30

(c)  
Initial cost of calculators
\[ \frac{16200}{27} = 600 \]

Discount offered as a percentage
\[ \frac{16200 - 16200}{600} \times 100 = 10\% \]

(10 marks)

Q20  
(a) (i) \(\begin{pmatrix} 1 \frac{-8}{2} \end{pmatrix} = \begin{pmatrix} 4 \ 2 \frac{1}{2} \end{pmatrix} \)

\[ N = \begin{pmatrix} -4,2 \frac{1}{2} \end{pmatrix} \]

\[ M = \frac{-8 + 12}{2}, 5 + \frac{-5}{2} \]
M is \((2,0)\)

(ii) \[
NM = \begin{pmatrix} 6 \\ -2 & 1/2 \end{pmatrix}
\]

\[
NM = \sqrt{6^2 + \left(-2 \frac{1}{2}\right)^2} = 6.5
\]

(b) \[
OB = \begin{pmatrix} 12 \\ -5 \end{pmatrix}, \quad NM = \begin{pmatrix} -6 \\ 2 & 1/2 \end{pmatrix}
\]

\[
\therefore \quad NM = \frac{1}{2} OB
\]

(c) \[
OP = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -6 \\ 2 & 1/2 \end{pmatrix}
\]

\[
OP^1 = \begin{pmatrix} -10 \\ 5 \end{pmatrix} + \begin{pmatrix} -5 \\ 8 \end{pmatrix} = \begin{pmatrix} -15 \\ 3 \end{pmatrix}
\]

\[
\therefore \quad P^1 \text{ is } (-15,13)
\]

Q21

(a) Volume of water

\[
\frac{6}{9+x} = \frac{2}{x} \Rightarrow x = 4.5
\]

\[
\therefore \quad \text{Vol} = \frac{1}{3} \times 3.142(6^2 \times 13.5 - 2^2 \times 4.5) = 490.152
\]

(b) (i) Volume of sphere

Top radius

\[
\frac{r}{14.5} = \frac{2}{4.5} = \frac{6}{13.5} \Rightarrow r = 6.4
\]

\[
\text{Vol} = \frac{1}{3} \times 3.142(6.444^2 \times 14.5 - 6^2 \times 13.5) = 121.6
\]

(ii) \[
\frac{4\pi r^3}{3} = 121.6
\]

\[
r^3 = 121.6 \times \frac{3}{4\pi}
\]

\[
r = 3.073
\]

(10 marks)

Q22
(a) \[ \frac{1}{2} \times 150h + \frac{1}{2} \times 200h + 300h = 10450 \]
\[ 475h = 10450 \]
\[ h = \frac{22 \times 60 \times 60}{1000} = 79.2 \text{km/h} \]

Max. speed = \[ \frac{22 \times 60 \times 60}{1000} = 79.2 \text{km/h} \]

(b) Acceleration = \[ \frac{22 \text{ m/s}}{150 \text{ s}} = \frac{11}{75} \text{ m/s}^2 \text{ or } 0.1467 \text{ m/s}^2 \]

(c) \[ \frac{1}{2} \times 100 \times 11 = 550 \]

(d) Time for half of journey
\[ \frac{1}{2} \times 22(150 + t + t) = \frac{1}{2} \times 10450 \]
\[ t = 162.5 \]

Total time = 150 + 162.5 = 312.5

(10 marks)
Q23

(a) Direction and distance of Q and P
   Direction and distance of R and P

(b) (i) Distance conversion
     \[8.5 \times 40 = 340\]

   (ii) North line at Q
        Bearing 063°

(c) (i) Distance from top of post at Q to top of post at P
      \[x = \frac{240}{\cos 9^\circ} \quad \text{or} \quad x \cos 9^\circ = 240\]
      \[= 243 \text{ m}\]

   (ii) Speed of bird
        \[\frac{243 \times 60 \times 60}{100 \times 18} \approx 48.6 \text{ km/h}\]

(10 marks)

Q24 (a)

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = \frac{1}{2} x^2 - x + 3)</td>
<td>3</td>
<td>2½</td>
<td>3</td>
<td>4½</td>
<td>7</td>
<td>10½</td>
<td>15</td>
</tr>
</tbody>
</table>
(b) \[ y_1 = \frac{1}{2} \times 1.5^2 - 1.5 + 3 - 2.625 \]

\[ y_2 = \frac{1}{2} \times 2.5^2 - 2.5 + 3 - 3.625 \]

\[ y_3 = \frac{1}{2} \times 3.5^2 - 3.5 + 3 - 5.625 \]

\[ y_4 = \frac{1}{2} \times 4.5^2 - 4.5 + 3 - 8.625 \]

\[ y_5 = \frac{1}{2} \times 5.5^2 - 5.5 + 3 - 12.625 \]

Approximate area
\[ \int (2.625 + 3.625 + 5.625 + 8.625 + 12.625) \]
\[ 33125 \text{ sq. units} \]

\[ \text{Area} = \int \left( \frac{1}{2} x^2 - x + 3 \right) dx = \left[ \frac{x^3}{6} - \frac{x^2}{2} + 3x \right]_{6}^{6} \]

\[ \left[ \frac{6^6}{6} - \frac{6^2}{2} + 3 \times 6 \right] - \left[ \frac{1^3}{6} - \frac{1^2}{2} + 3 \right] = 33.58 \]

\[ \% \text{ error} = \frac{33.58 - 33.125}{33.125} \times 100 \]

\[ = 0.625\% \]

(10 marks)
30.3.2 Mathematics Paper 2 (121/2)

Q1. 1 cow feed on \( \frac{480}{2 \times 4} \) kg in 1 day

\[ = 60 \text{ kg} \]

No. of cows to feed on 21060 kg in 6 weeks

\[ = \frac{20160}{60 \times 6 \times 7} \]

\[ = 8 \]

(3 marks)

Q2. \( (x - 1.5 - \sqrt{2}) (x - 1.5 + \sqrt{2}) = 0 \)

\[ x^2 - 1.5x + x\sqrt{2} - 1.5x + 2.25 - 1.5\sqrt{2} - x\sqrt{2} + 1.5\sqrt{2} - 2 = 0 \]

\[ 4x^2 - 12x + 1 = 0 \]

(3 marks)

Q3. \( M = c + kt^2 \)

\[ 40 = c + 4k \]

\[ 65 = c + 9k \]

\[ 25 = 5k, k = 5 \]

\[ 40 = c + 4 \times 5 \]

\[ c = 20 \]

When \( t = 4, M = 20 + 5 \times 16 \)

\[ = 100 \text{ kg} \]

(4 marks)

Q4
The average rate of change
\[ \frac{15.5 - 7.6}{3.4 - 1.8} = 4.9375^\circ \text{c/min} \]

Q6

\[ CO = -\frac{1}{3} \begin{pmatrix} 6 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \text{ or } OC = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \]

\[ AD = \frac{3}{4} \begin{pmatrix} 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \]

\[ CD = CO + OA + AD \]
\[ = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix} \]
\[ = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \]

(3 marks)
Q7 The LCM of 3 and 5 = 15
In 15 minutes, 8 customers will be served
\[ \therefore \text{total time} = \frac{200}{8} \times 15 \]
\[ 375 \text{ min.} \] (3 marks)

Q8 (a) \[
(2 - x)^7 = 2^7 - 7(2^6)x + 21(2^5)x^2 - 35(2^4)x^3 + 35(2^3)x^4 - 21(2^2)x^5 + 7(2)x^6
\]
\[ = 128 - 448x + 672x^2 - 560x^3 + 280x^4 - 84x^5 + 14x^6 - x^7 \]
(b) \[
(1.97)^3 = (2 - 0.03)^3 = 128 - 448(0.03) + 672(0.03)^2 - 560(0.03)^3 = 115.14968 \approx 115.1497 \] (4 marks)

Q9 Image area \[[(4 \times 2)-(5 \times 1)] \times 21\]
\[ = 63 \text{ sq. units} \] (3 marks)

Q10 \[
\frac{\sqrt{3}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3}(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = 3 + \sqrt{6} \] (2 marks)

Q11 \[(2 - 1)^2 + (5 - k)^2 = 10\]
k\[k^2 - 10k + 16 = 0\]
\[(k - 2)(k - 8) = 0\]
k\[k = 2 \text{ or } k = 8\]
Centre at \((1,2)\) or \((1,8)\) (3 marks)

Q12 \[
\left(\frac{1}{7} \times \frac{2}{5}\right) + \left(\frac{6}{7} \times \frac{1}{6}\right) = \frac{7}{35} \] (2 marks)

Q13 Longitude difference = \[45^0 + 60^0 = 105^0\]
Distance in km
\[\frac{105}{360} \times 2 \times 3.142 \times 6370 \cos 40^0\]
\[\frac{8943.7}{8943.7} \text{ km} \] (3 marks)

Q14 \[4 - 4 \cos^2 \alpha = 4 \sin \alpha - 1\]
\[4 - 4(1 - \sin^2 \alpha) = 4 \sin \alpha - 1\]
\[4 \sin^2 \alpha - 4 \sin \alpha + 1 = 0\]
402
\[(2 \sin \alpha - 1)(2 \sin \alpha - 1) = 0\]
\[
\sin \alpha = \sqrt{2} \quad \Rightarrow \quad \alpha = 30^0, 150^0
\]

Q15 \[AT^2 = 9 \times 4\]

\[= 36\]

\[\therefore AT = 6 \text{ cm}\]

(4 marks)

Q16 \[\int (3t^2 - 6t - 9) \, dt = t^3 - 3t^2 - 9t + c\]

\[
\left[ t^3 - 3t^2 - 9t \right]_0^4 = \left[ 4^3 - 3(4)^2 - 9(4) \right] - \left[ 0^3 - 3(0)^2 - 9(0) \right]
\]

\[= 7\]

Distance travelled = 16 + 7

\[= 23 \text{ m}\]

(2 marks)

Q17 (a) Total rate of flow in litres

\[120 \div 150 = 270 \text{ l/min}\]

Time taken = \[
\frac{18900}{270} = 70 \text{ min (1 hr 10 min)}\]

(b) (i) Part of tank filled after 25 min

\[270 \times 25 = 6750\]

Time taken to fill remaining part

\[
\frac{18900 - 6750}{270 - 20} = 48.6 \text{ min}\]

Total time to fill tank

\[25 + 48.6 = 73.6 \text{ min}\]

(ii) Total inflow into tank

\[270 \times 73.6 = 19872\]

Water wasted = 91872 - (542 \times 25 + 6300)

\[= 221\]

(10 marks)

Q18. (a) Value after 9 yrs = \[1240000 \left( 1 + \frac{12}{100} \right)^9\]

\[\approx 3438617.659\]

\[\approx 3438618\]

(b) (i) \[1240000(1.12)^9 = 2741245\]
\[ n \log 1.12 = \log \left( \frac{2741245}{1240000} \right) \]

\[ n = \frac{\log 2.210681452}{\log 1.12} \]

\[ n = 7 \]

(ii) \[ 1240000 \left( 1 + \frac{r}{100} \right)^7 = 2917231 \]

\[ 1 + \frac{r}{100} = \sqrt[7]{\frac{2917231}{1240000}} \]

\[ 1 + \frac{r}{100} = 1.130000011 \]

\[ r = 13\% \]

(10 marks)

(b)(i) Median goals = 25.5 ± 0.5

(ii) number of matches in which scores were between 0 & 37 = 49

(iii) \[ Q_1 = 19 ± 0.5 \]

\[ Q_3 = 33 ± 0.5 \]

Inter quartile Range 33-19=14

(10 marks)
Q20. (a) \[
\begin{bmatrix}
-0.6 & 0.8 \\
0.8 & 0.6
\end{bmatrix}
\begin{bmatrix}
5 & 10 & 10 \\
5 & 10 & 15
\end{bmatrix}
= \begin{bmatrix}
1 & 2 & 6 \\
7 & 14 & 17
\end{bmatrix}
\]

\(P'(1,7), Q'(2,14), R'(6,17)\)

(b)

(c) (ii) \[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\begin{bmatrix}
5 & 10 & 10 \\
5 & 10 & 15
\end{bmatrix}
= \begin{bmatrix}
-1 & -2 & -6 \\
7 & 14 & 17
\end{bmatrix}
\]

\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
= \begin{bmatrix}
0.6 & -0.8 \\
0.8 & 0.6
\end{bmatrix}
\]

(iii) Rotation about \((0,0)\) thro' angle \(53^0\)

\(10\) marks

Q21. (a) Tax on Kshs \(9680 = 9680 \times \frac{10}{100} = 968\)

Tax on Kshs \(18800 - 9680\) = \(9120 \times \frac{15}{100}\)

= 1368

Tax on Kshs \(24200 - 18800\) = \(5400 \times \frac{20}{100}\)

= 1080

Total tax = Kshs \((968 + 1368 + 1080)\)

= 3416
(b) \[ \text{Tax paid} = 3416 - \left( 1056 + 2400 \times \frac{15}{100} \right) = 2000 \]

(c) \[ \text{Increase in tax paid} = 2000 \times \frac{36.3}{100} = 726 \]
\[ \therefore \text{increase in earnings} = 726 \times \frac{100}{20} = 3630 \]
\[ \text{% increase} = \frac{3630}{24200} \times 100\% = 15\% \]

Q22. (a) \[ AC = \sqrt{(15\sqrt{2})^2 + (15\sqrt{2})^2} = 30\text{cm} \]
(b) \[ \tan \theta = \frac{8}{30} \]
\[ \theta = 14.93^\circ \]

(c) \[ \text{Pyramid height} = \sqrt{(17\sqrt{2})^2 - 15^2} = 18.79\text{cm} \]
\[ VO = 18.79 + 8 = 26.79\text{cm} \]

(d) \[ \tan \alpha = \frac{18.79}{7.5\sqrt{2}} \]
\[ \alpha = 60.55^\circ \]

Q23. (a)(i) \[ \frac{8}{2} \{ 2 \times 2 + (8 - 1)d \} = 156 \]
\[ d = 5 \]

(ii) \[ \frac{n}{2} \{ 2 \times 2 + (n - 1)s \} = 416 \]
\[ 5n^2 - n = 832 \]
\[ 5n^2 - n - 832 = 0 \]
\[ (5n + 64)(n - 13) = 0 \]
\[ n = 13 \]

(b)(i) \[ \text{1st three terms of the G.P; } a + 2d, a + 4d, a + 7d \]
These terms are; \( a + 6, a + 12 \) and \( a + 21 \)
\[ r = \frac{a + 12}{a + 6} = \frac{a + 21}{a + 12} \]
\[ (a + 12)^2 = (a + 6)(a + 12) \]
\[ a^2 + 24a + 144 = a^2 + 27a + 126 \]

\[ a = 6 \]
\[ \therefore \text{1}^{\text{st}} \text{term} = 6 + 6 = 12 \]

(ii) \[ r = \frac{6 + 12}{6 + 6} = \frac{3}{2} \]

\[ S_9 = 12 \left( \frac{3}{2} \right)^9 - 1 \]
\[ = \frac{3}{2} - 1 \]
\[ = 898.6 \text{ (to 4 sf)} \]

(10 marks)

Q24.

(a) (i) Scale
(ii) Plotting
(iii) Line of best fit

(b)(i) Average volume of ball bearing
\[ = \frac{133 - 108}{6 - 4} \]
\[ = 12.5 \]

(ii) \[ \frac{y - 133}{x - 6} = 12.5 \]
\[ y = 12.5x + 58 \]

407.
(c) Volume of water in cylinder is the volume of $y$ when $x = 0$

\[ y = 12.5 \times 0 + 58 \]

\[ = 58 \]  

(10 marks)